Parametric Side-Scattering by Waves Near the Hybrid Resonance Frequencies in Inhomogeneous Plasmas

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The temporal growth rates are calculated for side-scattering of light by modes near the upper and lower hybrid frequencies in a turbulent, inhomogeneous plasma.

The existence of absolute side-scattering instabilities in an unmagnetized plasma with linear density profile is now well-known [1, 2]. These instabilities can occur at about the same threshold pump intensity as the corresponding backscattering instabilities [3]. For side-scattering, both of the excited waves remain in the plasma and can result in further scattering or decay, and subsequent absorption by plasma particles. Side-scattering processes are thus important for laser fusion applications.

Besides the density gradient, in most plasmas an external or spontaneous magnetic field is usually present [4]. The role of the latter on the backscattering instabilities has been considered [5, 6] and found to be of importance. To our knowledge, side-scattering instabilities in magnetized plasmas with a linear density profile have not been investigated before. In this note, we consider side-scattering by waves near the upper and lower hybrid frequencies. The effect of long-wavelength turbulence [7] on these processes is also discussed.

Consider an ordinary electromagnetic wave $2 E_0 z \cos(k_0 x - \omega_0 t)$ propagating perpendicular to a constant magnetic field $B_0 \hat{z}$ in a plasma with a linear density profile $n(x) = n_0(1 + x/L)$, where L is the density gradient scale length. We introduce a weak background long-wavelength turbulence, which causes a stochastic wave-number mismatch $\delta\varkappa(x)$. The correlation of the latter is taken to be Gaussian, i. e., $\langle \delta\varkappa(x)\,\delta\varkappa(x') \rangle = \Delta^2 \exp\{-(x-x')^2/2 L_T^2\}$, where $\Delta^2 = \langle |\delta\varkappa|^2 \rangle$ is the amplitude, and L_T is the characteristic length of the correlation. The angular bracket denotes averaging over the phases of the random functions.

Since for side-scattering the scattered light is near its turning point and propagates almost perpendicular to the density gradient, the usual WKB approach [8] fails. One should thus deal with the two coupled second-order differential equations for the excited modes [1]. Accordingly, we average the coupling equations over the fluctuations, combine these equations and obtain [6, 9] for the averaged electric field amplitude $\langle E_s \rangle$ of the scattered radiation ($\omega_s = \omega - \omega_0$, $\boldsymbol{k}_s = \boldsymbol{k} - \boldsymbol{k}_0 \approx k_{sy}\hat{y}$) the equation

$$[c^2\, d^2/dx^2 - c^2 k_{sy}^2 + \omega_s{}^2 - \omega_{pe}^2\,(1 + x/L)]\,\langle E_s \rangle = \frac{(1+\chi_i)\,\omega_{pe}^2\, k^2 v_0{}^2 \exp\{-R(x)\}\langle E_s \rangle}{(\omega^2 - \Omega_e{}^2)\,(1 + \chi_e + \chi_i)}\,, \tag{1}$$

where $\omega_{pj}^2 = 4\pi n_0 e^2/m_j$, $\Omega_j = e_j B_0/m_j c$, $j = e, i$, $v_0 = e E_0/m_e \omega_0$,

and $R(x) = \Delta^2 L_T{}^2\,[\pi^{1/2}(x/\sqrt{2} L_T)\,\mathrm{erf}(x/\sqrt{2} L_T) + \exp\{-x^2/2 L_T{}^2\} - 1]$,

whereas $\chi_{e,i}(\omega, k)$ are the low frequency electron and ion susceptibilities in an inhomogeneous magnetized plasma. We note that in the side-scattering problem for wave propagation perpendicular to the magnetic field, only the low frequency waves are affected by the latter. The scattered light is assumed to be also an ordinary mode. This corresponds to two-dimensional scattering [3].

First, we consider the case of scattering by waves near the upper hybrid resonance frequency. Neglecting ion motion, letting

$$\chi_e = -\omega_{pe}^2 (1 + x/L)/(\omega^2 - \Omega_e{}^2),$$

and transforming the independent variable such that

$$\varrho = x(\omega_{pe}^2/c^2 L)^{1/3} - (D + B)/2,$$

we obtained for $x \ll \Delta^{-1}, L_T$ from Eq. (1)

$$\left[\frac{d^2}{d\varrho^2} - \frac{q(\varrho^2 - \varrho_t{}^2)}{\varrho + a/q + B(1-q)/q}\right]\langle E_s \rangle = 0\,, \tag{2}$$

where

$$a = (D - B)/2\,,$$
$$D = (\omega_{pe}^2/c^2 L)^{-2/3}(\omega_s{}^2/c^2 - \omega_{pe}^2/c^2 - k_{sy}^2)\,,$$
$$B = (\omega_{pe}^2/c^2 L)^{-2/3}(\omega^2 - \omega_H{}^2)/c^2\,,$$
$$\omega_H{}^2 = \omega_{pe}^2 + \Omega_e{}^2\,,$$
$$\varrho_t{}^2 = (B + D)^2/4 q^2 + (\lambda - D B)/q\,,$$
$$\lambda = v_0{}^2 k^2 L^{4/3}/\omega_{pe}^{2/3} c^{4/3}\,,$$
$$q = 1 + \Delta^2(\omega_{pe}^2/c^2 L)^{-2/3}\lambda\,.$$

For $|\varrho| \leqq \varrho_t \ll a/q + B(1-q)/q$, Eq. (2) is a parabolic cylinder equation with two turning points at $\varrho = \pm\varrho_t$. The stability behavior is obtained from the corresponding eigenvalue equation [1].

$$a^2 + B D(1-q) + q\lambda = (2l + 1) \cdot q[a + B(1-q)]^{1/2}\,, \tag{3}$$

where l is a positive integer.

Assuming $\varepsilon = (v_0 k L \Delta/\omega_{pe})^2 \ll 1$ but $\lambda \gg 1$, we solve Eq. (3) by iteration. Letting $\omega = \omega_H - i\gamma$, where γ is the growth rate, we obtain

$$\gamma = -(\nu_1 + \nu_2) + \gamma_0 \left\{ 1 - \frac{(2l+1)\, c\, \omega_{pe}^{1/2}}{(2 v_0 k)^{3/2} L} \left[1 + \varepsilon \frac{\omega_0 - \omega_{pe} + 2\omega_H}{\omega_0 - \omega_{pe} + \omega_H} \right] + \frac{\varepsilon}{2} \left[1 + \frac{\omega_H(\omega_H - \omega_0)}{(\omega_0 - \omega_{pe} + \omega_H)^2} \right] \right\}, \quad (4)$$

where $\gamma_0 = v_0 k\, \omega_{pe}/(\omega_0 - \omega_{pe} + \omega_H)$, and $c^2 k^2 = 2 c^2 \mathbf{k} \cdot \mathbf{k}_0 + \omega_H{}^2 - 2\omega_0 \omega_H$.

The frequency shift caused by the pump has been neglected. Here, we have included the linear damping rates ν_1 and ν_2 of the two excited waves in a phenomenological manner. The condition $\varrho_t \ll a/q + B(1-q)/q$ corresponds to

$$v_0/c \gg (2l+1)^{2/3} (\omega_{pe}/c k)^{1/3} (k L)^{-2/3} q^{1/6}. \quad (5)$$

This inequality can be satisfied if the plasma is unstable.

On the other hand, the condition

$$\Delta[\varrho_t + (D + B)/2]\, (c^2 L/\omega_{pe}^2)^{1/3} \ll 1$$

turns out to be not stringent for our purpose.

As a special case, we consider the situation when the scattered wave is not propagating. This happens at the point where $\omega_0 = \omega_{pe} + \omega_H$, and is analogous to the quarter-critical density point [3] in an unmagnetized plasma. Accordingly, $k_{sy} = 0$ and $k = k_0$. The growth rate is then

$$\gamma = -(\nu_1 + \nu_2) + \frac{v_0 k_0 \omega_{pe}}{2\omega_H} \left[1 - \frac{(2l+1) c\, \omega_{pe}^{1/2}}{(2 v_0 k)^{3/2} L} (1 + 1.5\,\varepsilon) + \frac{\varepsilon}{2} (1 - \omega_{pe}/4\omega_H) \right], \quad (6)$$

where $c^2 k_0{}^2 = \omega_H{}^2 + 2\omega_{pe}\omega_H$. We note that for $B_0 = \Delta = 0$, Eq. (6) reduces to that obtained by Drake and Lee [3].

Next, we investigate side-scattering by modes near the lower hybrid frequency. These modes can be considered to be electrostatic as the frequencies are close to the resonance frequency. For

$$\Omega_i < \omega \approx |\Omega_e \Omega_i|^{1/2} < \Omega_e < \omega_{pe,i},$$

one has

$$\chi_j = -\omega_{pj}^2 (1 + x/L)/(\omega^2 - \Omega_j{}^2).$$

In this case, the growth rate can readily be obtained from the resulting parabolic cylinder equation following the method used above. For small but finite turbulence level and inhomogeneity, we find

$$\gamma \approx m_e k^2 v_0{}^2 \Delta L / m_i |\Omega_e \Omega_i|^{1/2}. \quad (7)$$

It should be pointed out that in the limit of an inhomogeneous non-turbulent plasma, Eq. (1) becomes an Airy equation. Hence the analysis leading to Eq. (7), which is based on the existence of two turning points in the equation, breaks down. However, in this case, other effects, such as the nonuniform expansion velocity of the plasma [1], cannot be neglected and the resulting equation is again a parabolic cylinder equation. This problem can then be analyzed similarly as above.

In conclusion, we have obtained by means of simple models the effect of external magnetic fields and weak turbulence on parametric side-scattering instabilities. We have shown that growth rates of the averaged amplitudes are reduced in the presence of an external magnetic field. On the other hand, long-wavelength turbulence enhances the inhomogeneous growth rates.

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